Abstract—Equipping base stations (BSs) with very large antenna arrays is a promising way to increase the spectral and energy efficiency of mobile communication systems without the need for new cell sites. However, the prominently theoretical works on this topic are based on several crucial assumptions about the wireless channel which have not been sufficiently validated by measurements. In this paper, we report on an outdoor measurement campaign with a scalable virtual antenna array consisting of up to 112 elements. The large amount of acquired data allows us to study several important aspects of large-scale MIMO systems. For example, we partially confirm the theoretical results based on uncorrelated channels which predict that the channels at different positions become more and more orthogonal as the number of antennas grows. However, for the measured channels, the marginal gain of an additional antenna quickly diminishes. Nevertheless, our results indicate that most of the theoretical benefits of large-scale MIMO could be realized also over the measured channels.

I. INTRODUCTION

The use of very large antenna arrays at the base stations (BSs) has been recently advocated as an effective means to significantly increase the capacity of cellular communication systems while possibly reducing their energy consumption [1], [2], [3]. The beauty of this approach is its simplicity: no new cell sites are required, no technical changes need to be made at the terminals, and simple linear precoders are close to optimal. However, these advantages of large-scale or “massive” MIMO systems come only into play under favorable propagation characteristics of the radio channel [4]. In short, “favorable” means here that the channel vectors between different user terminals (UTs) become orthogonal as number of BS-antennas grows. It is unclear though, if this is the case for real channels in urban environments.

The literature on channel measurements with large antenna arrays is still very limited. The authors of [5] report on some experiments with a cylindrical indoor antenna array with 128 elements. They consider outdoor-to-indoor transmissions and observe that the correlation between the channels cannot be arbitrarily reduced by increasing the number of antennas. Already from 20 antennas on, there is hardly any advantage from the use of additional antennas. A possible explanation for this is that the physical channel offers only a limited number of physical directions or degrees of freedom which cannot be increased by adding transmit antennas. This gives rise to the conclusion that the effectiveness of large-scale MIMO strongly depends on the radio environment. Another measurement campaign of outdoor-to-outdoor channels with a very large virtual linear antenna array (128 elements, λ/2-spacing, 7.3 m width) is described in [6]. Interestingly, the authors observe large-scale fading and a varying angular power spectrum along the antenna elements and argue that the wide-sense stationary assumption for such channels does not hold. Also first large-scale MIMO prototypes appear. In [7], a scalable hardware architecture based on an FPGA platform is described and some measurement results as well as implementation aspects are discussed. The last references indicate that the research on large-scale MIMO has left the purely theoretical stage. It is now important to validate if the theoretical performance gains can be realized in practice.

In this paper, we describe a recently developed test-bed for outdoor channel measurements with very large antenna arrays and discuss some of our first measurement results. In particular, we study to which extent the performance predictions based on independent and identically distributed (i.i.d.) channel vectors hold for real channels. We then analyze several other aspects such as the correlation coefficient of the channel vectors at different positions and the condition number of the channel matrix. In summary, our results indicate that despite fundamental differences between the i.i.d. and the measured channels, a large fraction of the theoretical performance could be achieved also in practice.

II. MEASUREMENT SETUP

The channel measurements were conducted outdoors on the Alcatel-Lucent campus in Stuttgart, Germany. For a graphical illustration, see Fig. 1. The setup consisted of a transmit antenna array mounted on the top of a large building at a height of 20 m (shown on the left) and two mobile single-antenna
receivers which were placed at 2 m distance from each other on top of a car. Measurements were taken at 30 different positions throughout the campus at a carrier frequency of 2.6 GHz with a bandwidth of 20 MHz, corresponding to 1200 active sub-carriers (15 kHz spacing). Only every 3rd sub-carrier was considered for the evaluation, leading to a total of 400 channel coefficients per antenna and measurement position. The measurement positions were selected to provide a good mix of different channel conditions (line-of-sight (LOS), non-LOS (NLOS), shadowing from buildings) which can be considered as representative for a residential urban area. The transmit antenna consists of a vertical array of 8 custom-built, dual polarized elements with \( \lambda/2 \) spacing (only one polarization direction was used). Seven of these elements can be rotated on a circle of 1 m radius to simulate a large cylindrical antenna array (see Fig. 2). The remaining antenna element is fixed (at sufficient distance from the array to avoid reflections) and only used to estimate the possible carrier phase offset between different positions. This setup was used to obtain channel measurements for a “virtual” antenna array with 112 elements, corresponding to 16 different angular positions (see Fig. 3). The angular separation between two adjacent positions was chosen to be 3.5°, which corresponds to a horizontal spacing of 6.1 cm, slightly more than half of the wavelength.

At each measurement point on the campus, the channel vectors corresponding to each of the 16 angular positions were successively recorded. Since the campus is rather quiet without significant traffic or moving objects, the channel between the different measurements can be assumed to be quasi-static. The result of this measurement campaign is essentially a radio channel map of the campus (2 × 30 = 60 different locations) with an antenna array of 112 elements, taken over 400 sub-carriers. Thus, the available data is sufficient to calculate averages over different positions as well as averages over different sub-carriers.

![Fig. 2. Schematic of the transmit antenna array. One antenna element remains fixed while 7 antenna elements with \( \lambda/2 \) vertical spacing can be rotated to different angular positions.](image)

![Fig. 3. Top view of the rotating antenna array. Channel measurements were taken successively for 16 fixed angular positions to emulate a cylindrical array of 112 antennas.](image)

### III. Evaluation

The acquired data allows for the study of various aspects of large MIMO systems. First, we will compare the measured channels against randomly created channels with i.i.d. entries in terms of their capacity and achievable sum-rates with linear precoders. Second, we will study the orthogonality between the channel vectors at different measurement positions by various metrics, namely the correlation coefficient and the inverse condition number.

#### A. Capacity and achievable rates with linear precoding

We denote by \( \mathbf{h}_{i,f,N} \in \mathbb{C}^N \) and \( \mathbf{h}_{j,f,N} \in \mathbb{C}^N \) the channel vectors on sub-carrier \( f \) at positions \( i \) and \( j \), respectively, when only the first \( N \) of the available 112 antennas are used. These \( N \) antennas are selected at random and we average over different choices. This is done to avoid correlation effects between neighboring antennas\(^1\) which is an important aspect to study in its own regard but which is not considered in this paper. The channel vectors at each position are normalized such that the average received power over all antennas and sub-carriers is equal to one. We define the channel matrix

\[
\mathbf{H} = [\mathbf{h}_{i_1,f,N} \ldots \mathbf{h}_{i_K,f,N}] \in \mathbb{C}^{N \times K}
\]

which is constructed from the channel vectors \( \mathbf{h}_{i_k,f,N} \) at \( K \leq N \) randomly chosen measurement positions \( i_1, i_2, \ldots, i_K \). For notational simplicity, we ignore the dependence of \( \mathbf{H} \) on \( N, K, f, \) and the explicit positions \( i_k, k = 1, \ldots, K \). If we assume that the BS transmits data simultaneously to \( K \) user terminals (UTs), the vector \( \mathbf{y} \in \mathbb{C}^K \) of the received signals is given as

\[
\mathbf{y} = \sqrt{\text{SNR}} \mathbf{H}^H \mathbf{s} + \mathbf{n}
\]

where \( \mathbf{s} \in \mathbb{C}^K \) is the transmit vector satisfying \( \mathbb{E} [\mathbf{s}^H \mathbf{s}] = 1 \), \( \text{SNR} \) is the transmit signal-to-noise-ratio (SNR), and \( \mathbf{n} \sim \)

\(^1\)Especially, the correlation between horizontal and vertical neighboring antennas it not the same due to a different angular spread in both dimensions.
\( \mathcal{C}(0, I_K) \) is a vector of complex Gaussian noise. The sum-capacity \( C_{\text{sum}} \) of this channel is given as, e.g., [8],

\[
C_{\text{sum}} = \max_{Q \geq 0, \text{tr } Q \leq \text{SNR}} \log_2 \det \left( I_N + H^H Q H \right) \tag{2}
\]

which can be computed by a simple water-filling algorithm over the eigenvalues of \( H^H H \) [9] and is achieved by the non-linear dirty-paper coding strategy. Less optimal, but also less complex are linear beamforming strategies, where the transmitted signal \( s \) is given as

\[
s = \frac{1}{\sqrt{\text{tr } W W^H}} W x \tag{3}
\]

where \( W \in \mathbb{C}^{N \times K} \) is a precoding matrix and \( x \sim \mathcal{C}(0, I_K) \). The achievable sum-rate \( R_{\text{sum}} \) with linear precoding is given as

\[
R_{\text{sum}} = \sum_{k=1}^{K} \log_2 \left( 1 + \gamma_k \right) \tag{4}
\]

where

\[
\gamma_k = \frac{|h_{i,k,f,N}^H w_k|^2}{\text{SNR} + \frac{1}{K} \sum_{k=1}^{K} |h_{i,k,f,N}^H w_k|^2} \tag{5}
\]

and \( W_{[k]} \) is matrix \( W \) with its \( k \)th column \( w_k \) removed. We consider two different types of precoding, namely minimum-mean-square-error (MMSE) precoding [10] and conjugate beamforming (BF). The precoding matrices for both schemes are respectively defined as

\[
W_{\text{MMSE}} = H \left( H^H H + \frac{K}{\text{SNR}} I_K \right)^{-1} \tag{6}
\]

\[
W_{\text{BF}} = H. \tag{7}
\]

From the last equations, it is easy to see that \( W_{\text{MMSE}} \) is simply a scaled version of \( W_{\text{BF}} \) if the columns of \( H \) are orthogonal, i.e., \( H^H H = I_K \). Thus, the more orthogonal the columns of \( H \), the closer is the sum-rate with conjugate BF \( R_{\text{BF}} \) to the sum-rate with MMSE precoding \( R_{\text{MMSE}} \).

In Figs. 4 and 5, we show the the capacity and sum-rates as a function of the SNR for \( N = 10 \) and \( N = 112 \) antennas, respectively. In both plots, we have set \( K = 8 \) and averaged over 400 different permutations of random positions \( i_1, \ldots, i_K \) and the 400 sub-carriers. For comparison purposes, we also depict all rates for i.i.d. channels, i.e., the channel vectors are i.i.d. as \( h_{i,f,N} \sim \mathcal{C}(0, I_N) \). We can make the following observations. For \( N = 10 \) antennas and at high SNR, there is a significant gap between \( C_{\text{sum}} \) and \( R_{\text{MMSE}} \) for the measured and the i.i.d. channels. However, for \( N = 112 \), this gap vanishes entirely for the i.i.d channels and becomes significantly smaller for the measured channels. This shows that linear MMSE precoding is close to optimal for a large number of antennas.\(^2\) The performance loss for \( R_{\text{MMSE}} \) between the i.i.d. and the measured channels is roughly 4 dB for \( N = 112 \) and stays almost constant for the entire range of SNR values. Concerning

\(^2\)Note that MMSE precoding is identical to zero forcing at high SNR.

the much simpler conjugate beamforming precoding strategy, we can see that the performance difference between the measured and the i.i.d. channels is much larger than for MMSE precoding. The main reason for this is that beamforming does not avoid multi-user interference. Consequently, the interference level depends alone on the correlation between different channel vectors. We can hence conclude that the measured channels are more correlated than the i.i.d. channels. This aspect will be discussed in more detail in the next section.

**B. Correlation coefficient**

We define the correlation coefficient \( \delta_{i,j}(f, N) \) between the two channel vectors \( h_{i,f,N} \) and \( h_{j,f,N} \) as

\[
\delta_{i,j}(f, N) = \frac{|h_{i,j,f,N}^H h_{j,f,N}|}{||h_{i,f,N}|| ||h_{j,f,N}||} \tag{8}
\]
where \( \| \cdot \| \) denotes the Euclidean norm. Clearly, \( 0 \leq \delta_{i,j}(N,f) \leq 1 \). A low correlation between the channels at two different positions is desirable as it would allow one to simultaneously serve different UTs with little cross-talk. If the elements of \( h_{i,f,N} \) are i.i.d., the correlation can be made arbitrarily small by increasing \( N \) [1], [2], i.e.,

\[
\delta_{i,j}^{\text{iid}}(f,N) \xrightarrow{\text{a.s.}} 0 \quad (9)
\]

where “a.s.” denotes almost sure convergence.

Our first goal is to verify if this assumption also holds for real channels. Fig. 6 shows the correlation coefficient \( \delta_{i,j}(f,N) \) averaged over 400 random pairs of measurement points \((i,j)\) and 400 sub-carriers as a function of the number of antennas \( N \). For comparison purposes we also depict the expected correlation coefficient for Rayleigh fading channels, i.e., \( h_{i,f,N} \sim \mathcal{CN}(0,1) \). In this case, it is easy to show that \( \mathbb{E}[(\delta_{i,j}^{\text{ray}}(f,N))^2] = N^{-1} \). While the correlation of the measured channels decreases equally fast as the correlation of the i.i.d. channels for small \( N \), it decreases at a much slower rate from \( N \geq 10 \) on. Note that a similar effect has been observed in [5]. Thus, additional antennas only help up to a certain degree to make the channels between two terminals more orthogonal. This effect depends of course on the radio environment.

C. Condition number

The correlation coefficient evaluates the “orthogonality” of the channel vectors at two measurement positions. However, the main purpose of large-scale MIMO systems is to simultaneously serve multiple UTs. Thus, it is even more important to study the joint orthogonality of multiple channel vectors. To this end, we consider the condition number \( \kappa_{N,K} \in [1, \infty) \) of the matrix \( \mathbf{H}^H \mathbf{H} \), i.e.,

\[
\kappa_{K,N} = \frac{\text{largest eigenvalue of } \mathbf{H}^H \mathbf{H}}{\text{smallest eigenvalue of } \mathbf{H}^H \mathbf{H}} \quad (10)
\]

where \( \mathbf{H} = [h_{i_1,f,N} \cdots h_{i_K,f,N}] \in \mathbb{C}^{N,K} \) is constructed from the normalized channel vectors \( \bar{h}_{i,f,N} = h_{i,f,N}/\|h_{i,f,N}\| \) at \( K \leq N \) randomly chosen measurement positions \( i_1, i_2, \ldots, i_K \). The condition number is a widely used indicator for the performance of linear receivers/precoders, see, e.g., [11], [12], [13]. A large condition number means that the columns of \( \mathbf{H} \) are strongly correlated while \( \kappa_{K,N} = 1 \) implies that all columns are orthogonal. In order to obtain a meaningful metric taking values in a finite range, we consider the inverse of the condition number \( \kappa_{N,K}^{-1} \in [0,1] \). As a consequence of (9), it follows for i.i.d. channels that

\[
\mathbf{H}^H \mathbf{H} \xrightarrow{\text{a.s.}} \mathbf{I}_K \quad (11)
\]

and hence \( \kappa_{N,K}^{-1} \rightarrow 1 \), almost surely, as \( N \rightarrow \infty \). We would like to remark that the condition number can also lead to misleading conclusions regarding the system performance. Even for rank deficient channels, i.e., the smallest eigenvalue is equal to zero, the capacity can still be high (depending on the remaining eigenmodes of the channel).

Fig. 7 shows the average inverse condition number \( \kappa_{N,K}^{-1} \) as a function of \( N \) for different numbers of simultaneously considered measurement positions \( K \in \{2, 4, 6\} \). For comparison, we also provide results for i.i.d. channels. When only two channel vectors are considered and \( N \) is small, there is little difference between \( \kappa_{N,K}^{-1} \) for the measured and the i.i.d. channels. However, this difference becomes more and more pronounced as \( K \) and/or \( N \) increase. This implies that there is a significant amount of correlation between the channel vectors at different positions. We can also see that the larger \( K \), the larger is the number of antennas which still lead to considerable improvements of the condition number.

To further elaborate on the impact of \( N \) and \( K \) on \( \kappa_{N,K}^{-1} \), we show in Figs. 8 and 9 the cumulative distribution function (CDF) of \( \kappa_{N,K}^{-1} \) for \( N = 16 \) and \( N = 112 \) antennas, respectively. The difference between the curves for the measured and the i.i.d. channels is rather small for \( N = 16 \). However, for \( N = 112 \) antennas, this picture changes. In particular, the CDF is spread out over the entire range [0,1] which indicates that there are several heavily correlated channel vectors which cannot be rendered orthogonal by the use of more antennas. Thus, user selection becomes a crucial aspect of large-scale MIMO systems.

IV. Conclusions

In this paper, we have described a setup for channel measurements with large antenna arrays and presented some of the first results of a recently conducted measurement campaign. These measurements are necessary to verify some of the fundamental assumptions behind the quickly growing body of theoretical works on large-scale MIMO systems. In particular, we have analyzed by different metrics (correlation coefficient,
condition number) to which extent channel orthogonality between different terminals can be established by scaling up the number of transmit antennas. Our results show that despite significant differences between the i.i.d. and the measured channels, a huge fraction of the theoretical performance gains of large antenna arrays could be achieved in practice.

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