Market share analysis between MNO and MVNO under brand appeal based segmentation

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Abstract—We are interested in this paper by the competition between MNO and MVNO on a population of heterogeneous customers, with different perceptions of operators offer. The MVNO buys resources from MNO at the wholesale price and offers an added value supposed to be better and at a lower price compared to the incumbent operator. In such a scenario, the rational customer behavior is to choose the virtual operator offer. However, we consider that the market is segmented into two different populations: the first segment is defined by customers attracted by the brand appeal of the MNO as the incumbent operator and the second segment of customers more sensitive to the added value proposed by the MVNO. Thus, the choice of customers will rather depend on their preferences and retail operators offers. We model this problem considering game theory tools and we distinguish three possible situations depending on customers’ preferences. For each situation, we calculate the market share of each of the operators and the Nash equilibrium given by the MNO’s wholesale price and the MVNO’s margin.

A mobile virtual network operator (MVNO) is, in the broadest sense of the term, an operator that provides mobile phone services without owning a radio spectrum license or a network infrastructure. For this purpose, the MVNO must contract with a mobile network operator (MNO) with frequencies and mobile network infrastructure. This contract allows the MVNO to buy wholesale resources from the MNO and to resell in retail to subscribers. Unlike ordinary resellers, the MVNO relies on its brand image and reputation gained in other business to sell its mobile service[1].

MVNO offers are usually proposed as a bundle including, in addition to the mobile service, an added value related to their core business, as bonus points, discounts on music CDs or theater tickets. Through its original core business, the MVNO can directly target a specific segment of the mobile market, in other words a niche. We refer for example to Virgin Mobile offers which target a relatively young “In” population. Thus, a market that can be segmented regarding customer preferences, make the substitutability of operators offers a key factor in competition in the mobile market.

Mobile operators are faced with strategic issues when it comes to decide whether to accommodate or to foreclose the entry of the MVNO in the market. [3] and [4] consider a model with two vertically integrated MNO and one MVNO competing in a downstream market. In [3] Bourreau and

Yu-Shan lo [4] studies a similar model while incorporating the demand-side investment. The objective is to study how the MVNO behaves when facing entrant of different abilities. Hence, in Yu-Shan Lo model, users’ preferences are represented by a quasi-linear utility that depend on the product differentiation and the flat fee, which in turn depends on the added value.

Another relevant question is should MNOs be enforced to open their networks to MVNOs or not (by the regulator for example). Indeed, the introduction of MVNOs aims to lower retail prices through competition and use the extra resources that are not used by the MNOs. In [5] authors show that enforcing the MNO to open its network to MVNOs has a negative effect due to the fact that intense competition reduces the return on investment of MNO and therefore reduces the incentive to invest in its network. The MNO is indeed alone to bear infrastructure costs. The authors also show that semi-collusion in the investment increases the social welfare (semi-collusion in the sense that the MVNO participates in the investment costs).

Hence, one can expect the difficulty to introduce the first MVNO and once the market is open more MVNO can enter the competition. The French market for example has experienced its MVNO revolution in July 2004, hosting the first MVNO Debitel. Virgin Mobile’s entry on this market took place in 2006 and the total number of MVNOs has finally reached forty in 2010[6]. These operators are beginning to attract more and more market share especially young population which is more sensitive to added value proposed by MVNOs.

We will focus in this paper on MNO behavior when facing a segmented downstream market where users have an
heterogeneous preferences and have a different sensitivity to operators offers. It can be the case, of a market where for example old users trust the incumbent operators (Orange, SFR, Bouygues) and where young users prefers the added value proposed by an MVNO like Virgin Mobile. In that case, an important policy question is whether an MNO is likely to offer access to its network on a voluntary basis.

I. A MODEL FOR CUSTOMERS PERCEPTIONS OF OFFERS BASED ON BRAND APPEAL AND ADDED VALUE

We are interested in the competition between MNO and MVNO over a population of heterogeneous customers, with different perceptions of operators offers quality (added value of each offer). The MVNO buys MNO resources at the wholesale price and offers an added value supposed to be better at a lower price than the MNO.

In such a scenario, the rational behavior of customers is to choose the virtual operator offer. However, we believe that customers, at least part of them, rely also on the brand appeal while making their choice. Thus, in this paper, we consider that the market is segmented into two populations of different customers: customers that are attracted, at least to a certain level, by the well established brand of the incumbent operator and customers that are more sensitive to the added value offered by the MVNO. Thus, the choice of customers will rather depend on their preferences and operators retail offers.

We consider that MNO and MVNO are caracterized by the following parameters:

- \( V_1 \) and \( V_2 \) model resp. the added value of the MNO and MVNO. Since MVNO is supposed to innovate on added value we will consider that \( V_1 < V_2 \).
- \( p_1 \) and \( p_2 \) model resp. the unitary price charged by MNO (resp. MVNO) to customers that choose its offer. We consider again that MVNO should propose lower prices in order to attract customers. Hence, we consider \( p_1 > p_2 \).
- \( I_1 \) and \( I_2 \), the brand appeal of the MNO and MVNO. As the MNO is the incumbent operator we consider that it is more attractive with respect to brand appeal, thus \( I_1 > I_2 \).
- \( w \) is the unitary whole price charged by the MNO to the MVNO. A rational behavior expected from the MVNO is to set its retail price at least to the wholesale price in order to ensure a positive benefit.

The market is composed of \( N \) customers. We consider that this market is segmented into two segments \( S_1 \) and \( S_2 \) where \( S_1 \) refers to customers that are more sensitive to brand appeal and thus more likely attracted by MNO offers. Whereas \( S_2 \) refers to customers that are more sensitive to added value and thus more likely attracted by MVNO offers. We denote \( N_j \) the market share of segment \( S_j \). Each customer is caracterized by two parameters \( \alpha \) and \( \beta \) where:

- \( \alpha \) is the customer perception of brand appeal. Indeed, as the market segmentation is defined by brand appeal perception, we will consider only two possible values for this parameter each related to the segment to which the customer belongs. This parameter is thus indexed by the related segment.
- \( \beta \) is customer perception of added value. In order to cope with lack of information about actual perception of added value, we will suppose a uniform distribution on a bounded interval \([0, \beta_{max}]\). However it is possible to consider other laws to capture different perceptions of added value on both segments. For simplicity sake \( \beta_{max} \) is supposed to be the same for both segments, but again we can consider different values which will slightly modify market share computation.

Each customer will choose to subscribe to the operator that maximizes his surplus. We are inspired by the original formulation of Mussa and Rosen [7] to express the surplus of each customer. Thus, the surplus of customer \( i \) regarding the offer made by operator \( j \) is modeled by the following amount:

\[ \beta \times V_j + \alpha_i \times I_j - p_j, \]

where \( j = 1 \) for the MNO and \( j = 2 \) for the MVNO.

Let us note however that a customer still have the choice to refuse to subscribe or to delay the subscription process if the offers are not interesting enough i.e the price is too high, the added value or the brand appeal are not sufficient to compensate the raise of the offer price. This can be captured by the fact that the surplus is negative. We will refer to that case by index 0 as none of the operator are chosen.

Taking into account the proposed model for capturing customers’ perceptions of brand appeal and offer added value, we will compute the market share between the MNO and the MVNO.

II. THE MARKET SHARE OF MNO/MVNO

Let us consider a customer of the first segment :

- Such a customer prefers not to have access to the service rather than having the MNO as provider iff: \( \alpha_1 I_1 + \beta V_1 - p_1 < 0 \) That is \( \beta < \frac{p_1 - \alpha_1 I_1}{V_1} \).
- The customer prefers not to have access to the service rather than having the MVNO as a provider iff: \( \alpha_1 I_2 + \beta V_2 - p_2 < 0 \) That is \( \beta < \frac{p_2 - \alpha_1 I_2}{V_2} \).
- The customer prefers having the MVNO rather than the MNO as a provider iff: \( \alpha_1 I_1 + \beta V_1 - p_1 < \alpha_1 I_2 + \beta V_2 - p_2 \) that is \( \beta < \frac{(p_1 - p_2) + \alpha_1 (I_2 - I_1)}{V_1 - V_2} \).

Let us sort this three thresholds:

\[ \beta_1 = \frac{p_1 - \alpha_1 I_1}{V_1}; \]
\[ \beta_2 = \frac{p_2 - \alpha_1 I_2}{V_2}; \]
\[ \beta_3 = \frac{(p_1 - p_2) + \alpha_1 (I_2 - I_1)}{V_1 - V_2}. \]

Indeed their order is very important when computing the market share. This order is defined by prices set by each operator and may change in function of them. We will see that different order are possible but some of them are not valid as they imply absurd situations. Let us note \( i \to j \) (or
symetrically $j \leftarrow i$ if a customer with a given sensitivity $\beta$ prefers $j$ among $i$ where $i,j \in \{0, \text{MNO,MVNO}\}$
- case 1: $\beta_1 \leq \beta_3 \leq \beta_2$ : is valid
  - for $\beta < \beta_1$ customer prefers not to join any provider
  - for $\beta_1 < \beta < \beta_2$ customer prefers to join the MNO.
  - for $\beta_3 < \beta < \beta_{\text{max}}$ customer prefers to join the MVNO.
- case 2: $\beta_1 \leq \beta_3 \leq \beta_2$ : is not valid. Indeed if $\beta_3 < \beta < \beta_2$ than there is a cycle $0 \rightarrow 1 \rightarrow 2 \rightarrow 0$ which is absurd.
- case 3: $\beta_2 \leq \beta_3 \leq \beta_1$ : is not valid. Indeed if $\beta_2 < \beta < \beta_1$ than there is a cycle $0 \leftarrow 1 \leftarrow 2 \leftarrow 0$
- case 4: $\beta_2 \leq \beta_1 \leq \beta_3$ is not valid. Indeed if $\beta_2 < \beta < \beta_1$ than there is a cycle $0 \leftarrow 1 \leftarrow 2 \leftarrow 0$ which is absurd.
- case 5: $\beta_3 \leq \beta_2 \leq \beta_1$ : is valid
  - for $\beta < \beta_2$ customer prefers not to join any provider
  - for $\beta > \beta_2$ customer prefers to join the MVNO.
- case 6: $\beta_3 \leq \beta_1 \leq \beta_2$ is not valid. Indeed if $\beta_1 < \beta < \beta_2$ than there is a cycle $0 \rightarrow 1 \rightarrow 2 \rightarrow 0$

Let us prove that it is sufficient to consider case 1 and case 5 by comparing $\beta_3$ and $\beta_2$. Indeed we have only two cases either $\beta_3 \geq \beta_2$ or $\beta_3 \leq \beta_2$. Let us suppose that $\beta_3 \geq \beta_2$ and prove that $\beta_3 \geq \beta_1$ necessarily.

$\beta_3 \geq \beta_2 \Rightarrow (p_1-p_2)+\alpha_1(I_2-I_1) \geq \frac{p_2-\alpha_1I_2}{V_2}$

That is: $V_2(p_1-p_2)+\alpha_1V_2(I_2-I_1) \leq (V_1-V_2)(p_2-\alpha_1I_2)$

By expanding and simplifying, we get: $V_2p_1-\alpha_1V_2I_1 \leq V_1p_2-V_1\alpha_1I_2$

After further simplification: $\frac{p_1-\alpha_1I_1}{V_1} \leq \frac{(p_2-\alpha_1I_2)}{V_2}$. That is :

$\beta_2 \leq \beta_3 \Rightarrow \beta_1 \leq \beta_2 \Rightarrow \beta_1 \leq \beta_3$ (1)

This relates to case 1 where $\beta_1 \leq \beta_2 \leq \beta_3$.

Now, let us suppose that $\beta_3 \leq \beta_2$ and prove that $\beta_3 \leq \beta_1$ necessarily

$\beta_3 \leq \beta_2 \Rightarrow (p_1-p_2)+\alpha_1(I_2-I_1) \leq \frac{p_2-\alpha_1I_2}{V_2}$

That is: $V_2(p_1-p_2)+\alpha_1V_2(I_2-I_1) \geq (V_1-V_2)(p_2-\alpha_1I_2)$.

By expanding and simplifying, we get: $V_2p_1-\alpha_1V_2I_1 \geq V_1p_2-V_1\alpha_1I_2$ After further simplification: $\frac{p_1-\alpha_1I_1}{V_1} \geq \frac{(p_2-\alpha_1I_2)}{V_2}$. That is:

$\beta_3 \leq \beta_2 \Rightarrow \beta_1 \geq \beta_2 \Rightarrow \beta_3 \leq \beta_1$ (2)

This relates to case 5 where $\beta_3 \leq \beta_2 \leq \beta_1$.

Now let us analyse the situation for segment $S_2$ taking into account the situation of segment $S_1$. Note that for segment $S_2$ the thresholds are defined with $\alpha_2$ instead of $\alpha_1$. Suppose that segment $S_1$ is in case 1. Intuitively, customers in segment $S_2$ may be either in case 1 or 5 depending on their sensitivity to added value. However if segment $S_1$ is already in case 5 then segment $S_2$ is definitely in the same case. Indeed, if customers giving priority to brand appeal choose the MVNO than definitely customers from $S_2$ will do so. Let us analyse this case more carefully.

First suppose that $S_1$ is in case 1 that is

$$\frac{p_1-\alpha_1I_1}{V_1} \leq \frac{p_2-\alpha_1I_2}{V_2} \leq \frac{(p_1-p_2)+\alpha_1(I_2-I_1)}{V_1-V_2}$$

. For simplicity sake, we will consider in the following:

$$a_1 = \frac{p_1}{V_1}; a_2 = \frac{p_2}{V_2}; a_3 = (\frac{p_1-p_2}{V_1-V_2});$$

$$b_1 = I_1/V_1; b_2 = I_2/V_2; b_3 = (I_1-I_2)/(V_1-V_2).$$

We have then for segment $S_1$:

$$a_1-b_1a_1 \leq a_2-b_2a_2 \leq a_3-b_3a_3.$$  

Now we will analyse which case is valid for segment $S_2$ by comparing $a_1-b_1a_2 \leq a_2-b_2a_2$ and $a_3-b_3a_2$. First, we have $a_i-b_ia_2 > a_i-b_1a_1$ for $i = \{1,2\}$ since $a_1 > a_2$. However $a_3-b_3a_2 < a_3-b_3a_1$.

Now given those constraints let us show which situations are valid for segment $S_2$. We will check if segment $S_2$ may be in case 1 too. This is depicted in figure 1.

![Fig. 1. Case 1-1](image)

This situation is valid if the situation of the market is such that $a_1-b_1a_1 \leq a_1-b_1a_2 \leq a_2-b_2a_2 \leq a_3-b_3a_2 \leq a_3-b_3a_1$. In such a situation segment $S_2$ will be in case 1 and thus some customers will choose the MNO and some of them will choose the MVNO. We will relate to this case in the following as case 1-1. Since both segments have customers eventually interested in both operators, One can expect that in such a case the proportion of customers choosing the MVNO in segment 2 is more important than customers from segment $S_1$. This is confirmed by figure 2.

Let us note that the proportion of customers who will not choose either the MNO or the MVNO is less important in segment $S_1$ which can be explained by the fact that the brand appeal is much more a compensation in that segment.

Now let us check if segment 2 may be in case 5 that
Fig. 2. Market share of operators

is all clients in that segment are only interested in the MVNO. Let us first note that we have $a_2 - b_2 \alpha_2 > a_2 - b_2 \alpha_1$ since $\alpha_1 > \alpha_2 > 0$, and whatever the order of $a_i - b_i \alpha_1$ compared to $a_i - b_i \alpha_2$ (for $i = 1$ or 3). It is valid for segment $S_2$ to be in case 2 as depicted in figure 3.

We will relate to this case in the following as case 1-5.

Fig. 3. Case 1-5

5. In such situation the MNO looses all customers in segment $S_2$. An important proportion of customers is lost as they do not choose either the MNO or the MVNO.

Now suppose that segment 1 is in case 2 that is all customers from $S_1$ choose the MVNO and let us show that so will be the case for customers from segment 2. As segment 1 is in case 2 then $a_2 - b_2 \alpha_2 \leq a_1 - b_1 \alpha_1$. Let us suppose by contradiction that segment 2 is not in case 2 that is $a_1 - b_1 \alpha_2 < a_2 - b_2 \alpha_2$. We have then:

$$a_2 - b_2 \alpha_1 + a_1 - b_1 \alpha_2 < a_1 - b_1 \alpha_1 + a_2 - b_2 \alpha_2$$

thus : $b_1 (\alpha_1 - \alpha_2) < b_2 (\alpha_1 - \alpha_2)$ which is a contradiction as $(\alpha_1 - \alpha_2)$ is positive and $b_1 \geq b_2$.

Hence both segment 1 and segment 2 are in case 2 and choose either the MVNO or nothing. We will refer to that case as 5-5.

A. Case 1-1

Let us remind that case 1-1 refer to the case where both MNO and MVNO are in case 1 that is when there are customers from both segments maybe interested in their respective offers. Now, we will compute the total demand of each operator. First, we will consider the demand for MNO from the first segment in Case 1:

$$D_{\text{MNO}}^1 = \frac{N_1}{\beta_{\text{max}}} \int_{\frac{p_1 - p_2}{1} + \alpha_1 (I_2 - I_1)}^{\frac{p_2 - p_1}{1} - \alpha_1 (I_2 - I_1)} d\beta$$

With $N_1 = RN$ and $N_2 = (1 - R)N$, and $0 \leq R \leq 1$ then:

$$D_{\text{MNO}}^1 = \frac{RN}{\beta_{\text{max}}}[\frac{(p_1 - p_2) + \alpha_1 (I_2 - I_1)}{V_1 - V_2} - \frac{p_1 - \alpha_1 I_1}{V_1}]$$

In a similar way we have for segment 2:

$$D_{\text{MNO}}^2 = \frac{N_2}{\beta_{\text{max}}} \int_{\frac{p_1 - p_2}{1} + \alpha_2 (I_2 - I_1)}^{\frac{p_2 - p_1}{1} - \alpha_2 (I_2 - I_1)} d\beta$$

By denoting :

$$C_1 = \frac{RN}{\beta_{\text{max}}} \left[\frac{(\alpha_1 - \alpha_2) (I_2 - I_1)}{V_1 - V_2} + \frac{(\alpha_1 - \alpha_2) I_1}{V_1}\right] +$$

$$\frac{N_\alpha_2 (I_2 - I_1)}{\beta_{\text{max}} (V_1 - V_2)} + \frac{N \alpha_2 (I_2 - I_1)}{\beta_{\text{max}} (V_1 - V_2)} p_1 \equiv \text{cst}$$

And after simplification we get:

$$D_{\text{MNO}} = \frac{N}{\beta_{\text{max}} (V_2 - V_1)} p_2 + C_1$$

In an another hand the demand received by the MVNO from segment 1:

$$D_{\text{MVNO}}^1 = \frac{N_1}{\beta_{\text{max}}} \int_{\frac{p_1 - p_2}{1} + \alpha_1 (I_2 - I_1)}^{\frac{p_2 - p_1}{1} - \alpha_1 (I_2 - I_1)} d\beta$$

In a similar way, the demand received by the MVNO from segment 2 is given by:

$$D_{\text{MVNO}}^2 = \frac{N_2}{\beta_{\text{max}}} \int_{\frac{p_1 - p_2}{1} + \alpha_2 (I_2 - I_1)}^{\frac{p_2 - p_1}{1} - \alpha_2 (I_2 - I_1)} d\beta$$

Hence the total demand on MVNO is given by:

$$D_{\text{MVNO}} = D_{\text{MVNO}}^1 + D_{\text{MVNO}}^2$$
The MVNO profit is given by:

$$D^{MVNO} = \frac{RN}{\beta_{max}} [\beta_{max} \left( \frac{(p_1 - p_2) + \alpha_1 (I_2 - I_1)}{V_1 - V_2} \right) + \frac{(1-R)N}{\beta_{max}} \left( \frac{(p_1 - p_2) + \alpha_2 (I_2 - I_1)}{V_1 - V_2} \right)]$$

By denoting:

$$C_2 = N \left[ \frac{N (I_2 - I_1)}{\beta_{max} (V_1 - V_2)} (R \alpha_1 + (1-R) \alpha_2) - \frac{N \beta_{max}}{\beta_{max} (V_1 - V_2)} (\omega + \mu) \right] \equiv \text{cst}$$

Then after simplification we get:

$$D^{MVNO} = C_2 + \frac{N}{\beta_{max} (V_1 - V_2)} p_2$$

Now, we will compute the profit for both operators under case 1.1, then define the optimal strategies for each. Given that the MVNO price is set while taking into account the wholesale price \( \omega \) and an (eventually positive) margin \( \mu \), then the MNO profit can be written as:

$$\pi^{MNO} = D^{MNO} \mu + D^{MVNO} \omega$$

$$= \left( \frac{N}{\beta_{max} (V_2 - V_1)} (\omega + \mu) + C_1 \right) p_1 + \left( C_2 + \frac{N}{\beta_{max} (V_1 - V_2)} (\omega + \mu) \right) \omega$$

$$\equiv \left( \frac{N \beta_{max} (V_2 - V_1)}{\beta_{max} (V_1 - V_2)} \right) \omega^2 + \left( \frac{N p_1 - N \mu}{\beta_{max} (V_2 - V_1)} + c_2 \right) \omega$$

In order to define the best response of the MNO to the MVNO strategy we derive its profit with respect to \( \omega \):

$$\frac{\partial \pi^{MNO}}{\partial \omega} = 0 \Rightarrow 2 N \frac{\beta_{max} (V_1 - V_2)}{\beta_{max} (V_2 - V_1)} \omega + \frac{N p_1 - N \mu}{\beta_{max} (V_2 - V_1)} + C_2 = 0$$

The optimal response of the MNO is given by the wholesale price \( \omega \) as function of \( \mu \):

$$\omega = \frac{(I_2 - I_1)(R \alpha_1 + (1-R) \alpha_2) - \beta_{max} (V_1 - V_2) + 2p_1 - \frac{1}{2} \mu}{2}$$

The MVNO profit is given by:

$$\pi^{MVNO} = D^{MVNO} \omega$$

$$= \left( C_2 + \frac{N}{\beta_{max} (V_1 - V_2)} (\omega + \mu) \right) \mu$$

$$= \left( C_2 + \frac{N \omega}{\beta_{max} (V_1 - V_2)} \mu + \frac{N}{\beta_{max} (V_1 - V_2)} \mu^2 \right)$$

Now, to define the best response of the MVNO to the MNO strategy, we derive its profit with respect to \( \mu \):

$$\frac{\partial \pi^{MVNO}}{\partial \mu} = 0 \Rightarrow \frac{2N}{\beta_{max} (V_1 - V_2)} \mu + \frac{N \omega}{\beta_{max} (V_1 - V_2)} + C_2 = 0$$

The optimal response of the MVNO is given by the margin \( \mu \) as function of \( \omega \):

$$\mu = \frac{(I_2 - I_1)(R \alpha_1 + (1-R) \alpha_2) - \beta_{max} (V_1 - V_2) + p_1 - \frac{1}{2} \omega}{2}$$

At the equilibrium, both \( \omega \) and \( \mu \) are best response to each other. After substitution of \( \mu \) in (3) by its value from equation (4) and after simplification we get:

$$\omega^* = \frac{(I_2 - I_1)(R \alpha_1 + (1-R) \alpha_2) - \beta_{max} (V_1 - V_2) + p_1}{3}$$

Similarly, after substitution of \( \omega \) in (4) by its value from equation (3) and after simplification we get:

$$\mu^* = \frac{(I_2 - I_1)(R \alpha_1 + (1-R) \alpha_2) - \beta_{max} (V_1 - V_2)}{3}$$

By comparing \( \mu^* \) and \( \omega^* \), we note that \( \omega^* - \mu^* = p_1 \) in this case. Given the constraints that \( \omega^*, \mu^* \geq 0 \) and \( \omega^* \leq p_1 \), then necessarily, \( \omega^* = p_1 \). Hence the unique Nash equilibrium in this game is given by \( (\omega^*, \mu^*) = (p_1, 0) \).

The MNO chooses a foreclosure strategy by setting a wholesale price equal to its retail price. Thus, if the market is such that the competition between operators covers both segments, then the MNO has more incentives to preserve its downstream market than to try to win on the wholesale market. MNO acts aggressively by setting the maximum wholesale price such that the MVNO cannot make a profit. MVNOs can not exist in such a market unless enforced by regulation.

**B. Case 1-5**

Now let us consider case 1-5 where MVNO is still active on both segments but MNO is only active in segment S1. We will compute the total demand for both operators.

Similarly to Case 1-1, the demand received by MNO and the demand received by the MVNO from segment 1 are given by:

$$D_1^{MNO} = \frac{RN}{\beta_{max}} \left( \frac{(p_1 - p_2) + \alpha_1 (I_2 - I_1)}{V_1 - V_2} - \frac{p_1 - \alpha_1 I_1}{V_1} \right)$$

$$D_1^{MVNO} = \frac{RN}{\beta_{max}} \left( \frac{(p_1 - p_2) + \alpha_1 (I_2 - I_1)}{V_1 - V_2} - \frac{p_1 - \alpha_1 I_1}{V_1} \right)$$

However, for the segment 2, the MNO does not attract any customers. Hence

$$D_2^{MNO} = 0$$

The demand received by the MVNO for segment 2:
with respect to $\omega$ given by the wholesale price $\omega$

Given profit is given by:

$$\pi = 1.5$$

Then define the optimal strategies for each. First, the MNO

Now, we will compute the profit for both operators under case

To define the best response of the MNO, we derive its profit

Hence, the total demand of the MNO is given by:

Where:

$$C_3 = \frac{RN}{\beta_{\text{max}}} \left[ \frac{p_1 + \alpha_1(I_2 - I_1)}{V_1 - V_2} - \frac{p_1 - \alpha_1I_1}{V_1} \right] \equiv \text{cst}$$

And the total demand on the MVNO is given by:

$$D_{\text{MVNO}} = C_3 - \frac{RN}{\beta_{\text{max}}(V_1 - V_2)p_2}$$

And the total demand on the MVNO is given by:

$$D_{\text{MVNO}} = C_4 - \frac{N(RN + \frac{(R-1)\beta_{\text{max}}}{V_1 - V_2}(\omega + \mu))p_1 + }{\beta_{\text{max}}(V_1 - V_2)(\omega + \mu)}$$

$$\pi_{\text{MNO}} = \left( \frac{N\beta_{\text{max}}}{\beta_{\text{max}} + R\alpha_1(I_1 - I_2) - p_1} + (1-R)\frac{\alpha_2}{V_2}(\omega + \mu) \right)$$

To define the best response of the MNO, we derive its profit

$$\omega = \frac{-\frac{\beta_{\text{max}}}{2} + \frac{rp_1}{2(V_1 - V_2)(\frac{R}{V_1 - V_2} + \frac{(R - 1)}{V_2})}}{2}$$

In another hand the MVNO profit is given by:

$$\pi_{\text{MVNO}} = \frac{N}{\beta_{\text{max}}(V_1 - V_2)}(\omega + \mu)$$

To define the best response of the MVNO, we derive its profit

$$\mu = \frac{-\beta_{\text{max}}C_4}{2N\left(\frac{R}{V_1 - V_2} + \frac{(R - 1)}{V_2}\right)} - \frac{1}{2}$$

At the equilibrium, both $\omega$ and $\mu$ are best response to each other. After substitution of $\mu$ in (6) by its value from equation

And similarly,

There is a unique Nash equilibrium $(\omega^*, \mu^*)$ in this case
given by (9) and (8). Both operators may reach that equilibri-

where MNO concentrates its effort on population which

sensitive to its brand appeal, by proposing appropriate offers

and allowing MVNO to access the market. We will see that

this is the unique Nash equilibrium where both operators

have actual customers and their profit is non null.

C. Case 5.5

In this case, in both segments the demand received by the

MNO is null. That is MNO the price set by the MNO is to

high, its profit will be limited to the wholesale price charged
to the MVNO. We have:

$$D_{\text{MNO}} = 0$$

The demand received by the MVNO from segment $S_1$ is:

$$D_{1\text{MVNO}} = \frac{RN}{\beta_{\text{max}}}\int_{\frac{R}{V_1 - V_2} + \frac{(R - 1)}{V_2}} d\beta$$

The demand received by the MVNO from segment 2 is:

$$D_{2\text{MVNO}} = \frac{(1-R)N}{\beta_{\text{max}}}\int_{\frac{R}{V_1 - V_2} + \frac{(R - 1)}{V_2}} d\beta$$

The total demand on the MVNO is given by:

$$D_{\text{MVNO}} = D_{1\text{MVNO}} + D_{2\text{MVNO}}$$

$$D_{\text{MVNO}} = \frac{N}{\beta_{\text{max}}(p_2 - \alpha_2)(\alpha_1 + R(\alpha_1 - \alpha_2))}$$
Since the MNO does not receive demand, its profit is given by:

$$\pi^{MNO} = D^{MVNO} \omega$$

$$= \frac{N \beta_{max}}{V_2} \left( \omega - p_2 - I_2(\alpha_2 + R(\alpha_1 - \alpha_2)) \right)_\omega$$

To define the best response of the MNO, we derive its profit with respect to \( \omega \).

We obtain the optimal response of the MNO given by the wholesale price \( \omega \) as function of \( \mu \):

$$\omega = \frac{V_2}{2} \left( \beta_{max} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) - \frac{\mu}{V_2} \right)$$

In the other hand, the MVNO profit is given by:

$$\pi^{MVNO} = D^{MVNO} \mu$$

$$= \frac{N \beta_{max}}{V_2} \left( \omega - p_2 - I_2(\alpha_2 + R(\alpha_1 - \alpha_2)) \right)_\mu$$

To define the best response of the MVNO, we derive its profit with respect to \( \mu \).

We obtain the optimal response of the MVNO given by the margin \( \mu \) as function of \( \omega \):

$$\mu = \frac{V_2}{2} \left( \beta_{max} - \frac{\omega}{V_2} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) \right)$$

To compute the Nash equilibrium \((\omega^*, \mu^*)\), we substitute (11) in (10) and get:

$$\omega^* = \frac{V_2}{3} \left( \beta_{max} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) \right)$$

Similarly, we substitute (10) in (11) and get:

$$\mu^* = \frac{V_2}{3} \left( \beta_{max} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) \right)$$

The only Nash equilibrium in the case 5-5 is given by: \((\omega^*, \mu^*)\) where:

$$\omega^* = \mu^* = \frac{V_2}{3} \left( \beta_{max} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) \right)$$

Note that, at the equilibrium, the margin \( \mu^* \) is equal to the wholesale \( \omega^* \). This means that profit of the MNO is equal the profit of the MVNO, since MNO charge have no direct customers and make profit by selling wholesale resources. The equilibrium reflects a situation where both operators are satisfied by their profit as they obtain similar output. Note also that \( \mu^* = \omega^* \) in this case is only a special case of the equation (9) in case 1-5 as \( R = 0 \) since the market share of the MNO is null.

Finally, we have to verify if this equilibrium can exist in our game, and what should be the retail prices \( p_1^* \) and \( p_2^* \) at the equilibrium. The existance of this equilibrium is verified if \( \omega^*, \mu^* > 0 \), which is the case.

The retail price of the MVNO at the equilibrium is given by:

$$p_2^* = \omega^* + \mu^* = \frac{2}{3} \left( \beta_{max} V_2 + I_2(\alpha_2 + R(\alpha_1 - \alpha_2)) \right)$$

To be in the case 5-5, the retail price of the MNO must be large enough so that even if some customers are attracted by the brand appeal of the MNO they prefer to choose the MVNO. This can be the case if:

$$\frac{1}{V_1} > \frac{2}{V_2} \left( \beta_{max} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) \right)_1 + \alpha_2 (I_1 - \frac{V_1}{V_2} I_2)$$

Since \( (I_1 - \frac{V_1}{V_2} I_2) > 0 \) and \( \alpha_1 > \alpha_2 \), we should have

$$p_1^* > \frac{2}{3} \left( \beta_{max} + \frac{I_2}{V_2} (\alpha_2 + R(\alpha_1 - \alpha_2)) \right)_1 + \alpha_1 (I_1 - \frac{V_1}{V_2} I_2)$$

Hence, if MNO wants to place itself in situation 5-5 he should announce a retail price that meets the condition (13). Its profits will be limited to the total wholesale price charged to the MVNO.

D. Brief discussion

We have distinguish three cases:

- Case 1-1: Where MNO acts aggressively by setting its retail price equal to the wholesale price. The MVNO is unlikely to accept this situation unless it is sponsored otherwise (advertisement for example).
- Case 5-5: where MNO sets a very high retail price so that he have no actual customers. Its profit is limited to the wholesale price charged to the MVNO. We can see this situation as the opposite extreme regarding case 1-1. Here the MNO conserves its brand image since he does not sell off its price and still recover similar profit using the MVNO. This might be an incentive to use MVNO as interface in markets where he cannot set high prices without altering its image.
- Case 1-5: Both operators can set appropriate retail prices (as shown previously) so that they share the market. MNO concentrates on the segment which is sensitive to its brand appeal and let the MVNO access the market. This is likely the most viable scenario.

III. CONCLUSION AND PERSPECTIVES

We have presented a competitive model operated by a mobile operator and a mobile virtual network operator, competing in a segmented market based on brand appeal. We have provided economic modeling of operator interactions and response of customers depending on their perception of the proposed offers. We have highlighted three possible market situations, two of them representing extreme situations where only one operator is really active and a situation where both
operators are active and we have computed the optimal prices for each situation. In further work, we explore different distributions of customer perception and their impact on the power balance between the incumbent and the virtual operators.

REFERENCES


