

LOW-COMPLEXITY LINEAR PRECODING FOR MULTI-CELL MASSIVE MIMO SYSTEMS

Abla Kammoun¹, Axel Müller², Emil Björnson^{2,3}, and Mérouane Debbah²

¹King Abdullah University of Science and Technology (KAUST), Saudi Arabia

²Alcatel-Lucent Chair, Supélec, France, and ³KTH Royal Institute of Technology, Sweden

ABSTRACT

Massive MIMO (multiple-input multiple-output) has been recognized as an efficient solution to improve the spectral efficiency of future communication systems. However, increasing the number of antennas and users goes hand-in-hand with increasing computational complexity. In particular, the precoding design becomes involved since near-optimal precoding, such as regularized-zero forcing (RZF), requires the inversion of a large matrix. In our previous work [1] we proposed to solve this issue in the single-cell case by approximating the matrix inverse by a truncated polynomial expansion (TPE), where the polynomial coefficients are selected for optimal system performance. In this paper, we generalize this technique to multi-cell scenarios. While the optimization of the RZF precoding has, thus far, not been feasible in multi-cell systems, we show that the proposed TPE precoding can be optimized to maximize the weighted max-min fairness. Using simulations, we compare the proposed TPE precoding with RZF and show that our scheme can achieve higher throughput using a TPE order of only 3.

Index Terms—Massive MIMO, linear precoding, low complexity, multi-cell systems, random matrix theory.

1. INTRODUCTION

One of the major challenges in multi-cell systems is dealing with inter-cell and intra-cell interference. Currently, the general trend is to deploy multiple antennas at the base stations (BSs), thereby enabling multi-user MIMO with flexible spatial interference mitigation [2]. User separation in the downlink is then performed using linear precoding. Unfortunately, the use of optimal precoding designs is far from being incorporated in current wireless standards such as LTE-Advanced [3]. This can be attributed to the fact that very accurate instantaneous channel state information (CSI) is required, which can be cumbersome to achieve in practice [4]. One can imagine that this becomes worse as the number of BS antennas, M , and the number of users, K , increase. Interestingly, this is not the case in reality. As $M \rightarrow \infty$ for a

fixed K , simple linear precoding, like maximum ratio transmission (MRT), is asymptotically optimal [5] and robust to CSI imperfections [6]. Nevertheless, MRT is not very appealing at practical values of M and K , since it does not actively suppress residual inter-user interference [7]. In fact, a precoding design based on both M and K growing large, with a fixed ratio, yields better massive MIMO performance [7]. In this regime, RZF precoding is near-optimal from a throughput perspective. However, it requires calculation of the inverse of the Gram matrix of the joint channel of all users, which is a non-trivial operation with a complexity scaling of MK^2 . Fortunately, the system performance is predictable in the large- (M, K) regime, where advanced tools from random matrix theory provide deterministic approximations of the achievable rates [7]. In light of these results, we proposed in [1] to solve the precoding complexity issues in single-cell systems using a new family of precoding schemes called *TPE precoding*. This family was obtained by approximating the matrix inverse in RZF precoding by a matrix polynomial. A similar approach was independently proposed in [8].

In this paper, we extend the TPE precoding from [1] to the scenario of multi-cell massive MIMO systems. A special focus is placed on the analysis of realistic characteristics, including user-specific channel covariance matrices, imperfect CSI and pilot contamination. The proofs of our results can be found in the extended version of this paper; see [9].

2. SYSTEM MODEL

We consider the downlink of a multi-cell system composed of $L > 1$ cells. Each cell consists of a M -antenna BS serving K single-antenna users. We assume a time-division duplex (TDD) protocol where the BS acquires instantaneous CSI in the uplink and uses it for the downlink transmission, exploiting channel reciprocity. The TDD protocols are synchronized across cells, so that pilot signaling and data transmission take place simultaneously. The received complex baseband signal at the m th user terminal (UT) in the j th cell is

$$y_{j,m} = \sum_{\ell=1}^L \mathbf{h}_{\ell,j,m}^H \mathbf{x}_{\ell} + n_{j,m} \quad (1)$$

where $\mathbf{x}_{\ell} \in \mathbb{C}^{M \times 1}$ is the transmit signal from the ℓ th BS and $\mathbf{h}_{\ell,j,m} \in \mathbb{C}^{M \times 1}$ is the channel vector from that BS to the

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m th UT in the j th cell, and $n_{j,m} \sim \mathcal{CN}(0, \sigma^2)$ is the additive circularly-symmetric complex Gaussian noise with variance σ^2 . The channel vectors are modeled as Rayleigh fading with

$$\mathbf{h}_{\ell,j,m} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\ell,j,m}) \quad (2)$$

where the family of covariance matrices $(\mathbf{R}_{\ell,j,m})_{\ell=1,j=1,m=1}^{L,L,K}$ satisfy the following conditions:

- Bounded norm: $\limsup_M \|\mathbf{R}_{\ell,j,m}\|_2 < +\infty, \forall \ell, j, m$;
- Trace scaling: $\liminf_M \frac{1}{M} \text{tr}(\mathbf{R}_{\ell,j,m}) > 0, \forall \ell, j, m$;
- Finite dimensional matrix space for $\mathbf{R}_{\ell,j,m}$: It exists a finite integer $S > 0$ and a linear independent family of matrices $\mathbf{F}_1, \dots, \mathbf{F}_S$ such that $\mathbf{R}_{\ell,j,m} = \sum_{k=1}^S \alpha_{\ell,j,m,k} \mathbf{F}_k$.

Note that these conditions are less restrictive than the one used in [10], where $\mathbf{R}_{\ell,j,m}$ was assumed to belong to a finite set of matrices. It is also in agreement with several physical channel models presented in the literature; for example, the one-ring model with user groups from [11]. This channel model considers a finite number G of groups which share approximately the same location and thus the same covariance matrix. Let $\theta_{\ell,j,g}$ and $\Delta_{\ell,j,g}$ be, respectively, the azimuth angle and the azimuth angular spread between the cell ℓ and the users in group g of cell j . Moreover, let d be the distance between two adjacent antennas (see Fig. 1 in [11]). Then, the (u, v) th entry of the covariance matrix $\mathbf{R}_{\ell,j,m}$ for users in group g is

$$[\mathbf{R}_{\ell,j,m}]_{u,v} = \frac{1}{2\Delta_{\ell,j,g}} \int_{-\Delta_{\ell,j,g} + \theta_{\ell,j,g}}^{\Delta_{\ell,j,g} + \theta_{\ell,j,g}} e^{jd(u-v) \sin \alpha} d\alpha. \quad (3)$$

We also assume that all BSs use Gaussian codebooks and linear precoding, such that the j th cell transmits the signal

$$\mathbf{x}_j = \sum_{m=1}^K \mathbf{g}_{j,m} s_{j,m} = \mathbf{G}_j \mathbf{s}_j \quad (4)$$

where $\mathbf{G}_j = [\mathbf{g}_{j,1}, \dots, \mathbf{g}_{j,K}] \in \mathbb{C}^{M \times K}$ is the precoding matrix and $\mathbf{s}_j = [s_{j,1}, \dots, s_{j,K}] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_K)$ is the vector containing the data symbols for UTs in the j th cell. The transmission at BS j is subject to a transmit power constraint

$$\frac{1}{K} \text{tr}(\mathbf{G}_j \mathbf{G}_j^H) = P_j \quad (5)$$

where P_j is the average transmit power per user in the j th cell. The received signal (1) can be thus expressed as

$$y_{j,m} = \sum_{\ell=1}^L \sum_{k=1}^K \mathbf{h}_{\ell,j,m}^H \mathbf{g}_{\ell,k} s_{\ell,k} + n_{j,m}. \quad (6)$$

A well known feature of large-scale MIMO systems is channel hardening, which implies that the effective useful channel $\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m}$ converges to its average value as $M \rightarrow \infty$. We decompose the received signal as

$$y_{j,m} = \mathbb{E}[\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m}] s_{j,m} + s_{j,m} (\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m} - \mathbb{E}[\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m}]) + \sum_{(\ell,k) \neq (j,m)} \mathbf{h}_{\ell,j,m}^H \mathbf{g}_{\ell,k} s_{\ell,k} + n_{j,m}$$

and assume, similar to [6, 7], that the receiver only knows the average channel gain $\mathbb{E}[\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m}]$, the average sum interference power $\sum_{\ell,k} \mathbb{E}[|\mathbf{h}_{\ell,j,m}^H \mathbf{g}_{\ell,k}|^2]$ and the variance of the noise σ^2 . By treating interference as worst-case Gaussian noise, the ergodic achievable rate of UT m in cell j is

$$r_{j,m} = \log_2(1 + \gamma_{j,m}) \quad (7)$$

where

$$\gamma_{j,m} = \frac{|\mathbb{E}[\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m}]|^2}{\sigma^2 + \sum_{\ell,k} \mathbb{E}[|\mathbf{h}_{\ell,j,m}^H \mathbf{g}_{\ell,k}|^2] - |\mathbb{E}[\mathbf{h}_{j,j,m}^H \mathbf{g}_{j,m}]|^2}. \quad (8)$$

2.1. Model of Imperfect Channel State information

Based on a TDD protocol, the channel is estimated in the up-link. In each cell, the UTs transmit mutually orthogonal pilot sequences, thereby allowing the BS to acquire CSI. Since the same set of orthogonal sequences is reused in each cell, the channel estimation is corrupted by inter-cell interference; this is called pilot contamination [6]. To estimate the channel corresponding to UT k in cell j , each BS correlates the received signal with the pilot sequence of that user. This results in the processed received signal

$$\mathbf{y}_{j,k}^{\text{tr}} = \mathbf{h}_{j,j,k} + \sum_{\ell \neq j} \mathbf{h}_{j,\ell,k} + \frac{1}{\sqrt{\rho_{\text{tr}}}} \mathbf{n}_{j,k}^{\text{tr}} \quad (9)$$

where $\mathbf{n}_{j,k}^{\text{tr}} \sim \mathcal{CN}(0, \mathbf{I}_M)$ and $\rho_{\text{tr}} > 0$ is the effective pilot SNR [7]. The MMSE estimate $\hat{\mathbf{h}}_{j,j,k}$ of $\mathbf{h}_{j,j,k}$ is

$$\hat{\mathbf{h}}_{j,j,k} = \mathbf{R}_{j,j,k} \mathbf{S}_{j,k} \mathbf{y}_{j,k}^{\text{tr}} = \mathbf{R}_{j,j,k} \mathbf{S}_{j,k} \left(\sum_{\ell=1}^L \mathbf{h}_{j,\ell,k} + \frac{1}{\sqrt{\rho_{\text{tr}}}} \mathbf{n}_{j,k}^{\text{tr}} \right)$$

where $\mathbf{S}_{j,k} = (\frac{1}{\rho_{\text{tr}}} + \sum_{\ell=1}^L \mathbf{R}_{\ell,j,k})^{-1}$. The estimated channel vectors at the j th BS to all UTs in its cell is denoted by

$$\hat{\mathbf{H}}_{j,j} = [\hat{\mathbf{h}}_{j,j,1}, \dots, \hat{\mathbf{h}}_{j,j,K}] \in \mathbb{C}^{M \times K}. \quad (10)$$

We also define $\Phi_{j,\ell,k} = \mathbf{R}_{j,j,k} \mathbf{S}_{j,k} \mathbf{R}_{j,\ell,k} \in \mathbb{C}^{M \times M}$ and note that $\hat{\mathbf{h}}_{j,j,k} \sim \mathcal{CN}(\mathbf{0}, \Phi_{j,j,k})$ is independent from the estimation error $\hat{\mathbf{h}}_{j,j,k} - \mathbf{h}_{j,j,k}$ since the MMSE estimator is used.

3. MULTI-CELL LINEAR PRECODING

3.1. Regularized zero-forcing precoding

For multi-cell systems, the optimal linear precoding is unknown under imperfect CSI and requires extensive optimization procedures under perfect CSI [4]. Therefore, only heuristic precoding schemes are feasible in multi-cell systems. RZF precoding is the state-of-the-art heuristic scheme in terms of

system throughput [4]. Using the notation of [7], the RZF precoding matrix used by the BS in the j th cell is

$$\mathbf{G}_j^{\text{rzf}} = \sqrt{K} \beta_j \left(\widehat{\mathbf{H}}_{j,j} \widehat{\mathbf{H}}_{j,j}^H + K \varphi_j \mathbf{I}_M \right)^{-1} \widehat{\mathbf{H}}_{j,j} \quad (11)$$

where the scalar parameter $\beta_j > 0$ is set to satisfy the power constraint in (5) and φ_j is a positive regularizing parameter.

Prior works have considered the optimization of the parameter φ_j in the single-cell case. This parameter provides a balance between maximization of the channel gain at each intended receiver (φ_j is large) and the suppression of inter-user interference (when φ_j is small). To the authors' knowledge, a closed-form optimization of the regularization parameter for multi-cell scenarios has thus far not been achieved. Hence, previous works have been restricted to the analysis of intuitive choices of the regularization parameter φ_j . In this context, the work in [7] considers massive MIMO performance of the RZF precoding in the large- (M, K) regime, where M and K tend to infinity such that

$$0 < \liminf \frac{K}{M} \leq \limsup \frac{K}{M} < +\infty. \quad (12)$$

3.2. Truncated Polynomial Expansion Precoding

The matrix multiplications and inversion in (11) gives RZF precoding an unfavorable complexity scaling of MK^2 [1]. Building on the concept of TPE used in our work in [1], we propose a new class of low-complexity linear precoding schemes also for the multi-cell case. The proposed precoding originates from the Cayley-Hamilton theorem which states that the inverse of a $M \times M$ matrix can be written as a weighted sum of its first M powers. A simplified precoding can then be obtained by considering only the first matrix powers. Assume that the j th BS employs a truncation order J_j , then the proposed TPE precoding matrix is given by

$$\mathbf{G}_j^{\text{TPE}} = \sum_{n=0}^{J_j-1} w_{n,j} \left(\frac{\widehat{\mathbf{H}}_{j,j} \widehat{\mathbf{H}}_{j,j}^H}{K} \right)^n \frac{\widehat{\mathbf{H}}_{j,j}}{\sqrt{K}}$$

where $\{w_{n,j}, j = 0, \dots, J_j - 1\}$ are the J_j scalar coefficients that are employed by the j th BS and the normalization by \sqrt{K} controls the energy of the precoding matrix. Note that while the RZF precoding has only a single regularization parameter, the proposed TPE precoding scheme offers a larger set of J_j design parameters. These coefficients define a parametrized class of precoding schemes ranging from MRT ($J_j = 1$) to RZF precoding, which is achieved at $J_j = \min(M, K)$ by selecting $w_{n,j}$ from the characteristic polynomial of $\frac{\widehat{\mathbf{H}}_{j,j} \widehat{\mathbf{H}}_{j,j}^H}{K} + \varphi_j \mathbf{I}_M$. The coefficients are optimized later in this paper. The complexity of TPE precoding scales only as MK [1].

4. ASYMPTOTIC PERFORMANCE ANALYSIS

We provide in this section an asymptotic performance analysis of the proposed TPE precoding. In particular, we show

that in the large (M, K) -regime, the SINR experienced by the m th UT served by the j th cell can be approximated by a deterministic term which depends only on the channel statistics. Before stating our main results, we cast the SINR expression (8) in a simpler form. Let $\mathbf{w}_j = [w_{0,j}, \dots, w_{J_j-1,j}]^\top$ and let $\mathbf{a}_{j,m} \in \mathbb{C}^{J_j \times 1}$ and $\mathbf{B}_{\ell,j,m} \in \mathbb{C}^{J_j \times J_j}$ be given by

$$[\mathbf{a}_{j,m}]_n = \frac{\mathbf{h}_{j,j,m}^H}{\sqrt{K}} \mathbf{V}_{n,j} \frac{\widehat{\mathbf{h}}_{j,j,m}^H}{\sqrt{K}}, \quad n \in [0, J_j - 1]$$

$$[\mathbf{B}_{\ell,j,m}]_{n,p} = \frac{1}{K} \mathbf{h}_{\ell,j,m}^H \mathbf{V}_{n+p+1,\ell} \mathbf{h}_{\ell,j,m} \quad n, p \in [0, J_\ell - 1]$$

where $\mathbf{V}_{n,j} = \left(\frac{\widehat{\mathbf{H}}_{j,j} \widehat{\mathbf{H}}_{j,j}^H}{K} \right)^n$. Then, the SINR experienced by the m th UT in the j th cell is

$$\gamma_{j,m} = \frac{|\mathbb{E}[\mathbf{w}_j^H \mathbf{a}_{j,m}]|^2}{\frac{\sigma^2}{K} + \sum_{\ell=1}^L \mathbb{E}[\mathbf{w}_\ell^H \mathbf{B}_{\ell,j,m} \mathbf{w}_\ell] - |\mathbb{E}[\mathbf{w}_j^H \mathbf{a}_{j,m}]|^2}. \quad (13)$$

As $\mathbf{a}_{j,m}$ and $\mathbf{B}_{\ell,j,m}$ are of finite dimensions, it suffices to determine an asymptotic approximation of the expected value of each of their elements. For that, we introduce the functionals

$$X_{j,m}(t) = \frac{1}{K} \mathbf{h}_{j,j,m}^H \boldsymbol{\Sigma}(t, j) \widehat{\mathbf{h}}_{j,j,m}$$

$$Z_{\ell,j,m}(t) = \frac{1}{K} \mathbf{h}_{\ell,j,m}^H \boldsymbol{\Sigma}(t, \ell) \mathbf{h}_{\ell,j,m}$$

where $\boldsymbol{\Sigma}(t, j) = \left(\frac{t \widehat{\mathbf{H}}_{j,j} \widehat{\mathbf{H}}_{j,j}^H}{K} + \mathbf{I}_M \right)^{-1}$. It is easy to see that

$$[\mathbf{a}_{j,m}]_n = \frac{(-1)^n}{n!} X_{j,m}^{(n)} \quad (14)$$

$$[\mathbf{B}_{\ell,j,m}]_{n,p} = \frac{(-1)^{n+p+1}}{(n+p+1)!} Z_{\ell,j,m}^{(n+p+1)}. \quad (15)$$

where $X_{j,m}^{(n)}$ and $Z_{\ell,j,m}^{(n+p+1)}$ represent the derivatives of $X_{j,m}(t)$ and $Z_{\ell,j,m}(t)$ at $t = 0$.

Theorem 1 *Let $\bar{X}_{j,m}(t)$ and $\bar{Z}_{\ell,j,m}(t)$ be*

$$\bar{X}_{j,m}(t) = \frac{\delta_{j,m}(t)}{1 + t \delta_{j,m}(t)}$$

$$\bar{Z}_{\ell,j,m}(t) = \frac{1}{K} \text{tr}(\mathbf{R}_{\ell,j,m} \mathbf{T}_\ell(t)) - \frac{t \left| \frac{1}{K} \text{tr} \boldsymbol{\Phi}_{\ell,j,m} \mathbf{T}_\ell(t) \right|^2}{1 + t \delta_{\ell,m}(t)}$$

where for each $j = 1, \dots, L$, $m = 1, \dots, K$, $\delta_{j,m}(t)$ are the unique positive solutions to the following system of equations:

$$\delta_{j,m}(t) = \frac{1}{K} \text{tr} \left(\boldsymbol{\Phi}_{j,j,m} \left(\frac{1}{K} \sum_{k=1}^K \frac{t \boldsymbol{\Phi}_{j,j,k}}{1 + t \delta_{j,k}(t)} + \mathbf{I}_M \right)^{-1} \right)$$

and $\mathbf{T}_j(t) = \left(\frac{1}{K} \sum_{k=1}^K \frac{t \boldsymbol{\Phi}_{j,j,k}}{1 + t \delta_{j,k}(t)} + \mathbf{I}_M \right)^{-1}$. In the large- (M, K) regime we have

$$\mathbb{E}[X_{j,m}(t)] - \bar{X}_{j,m}(t) \xrightarrow{M, K \rightarrow +\infty} 0$$

$$\mathbb{E}[Z_{\ell,j,m}(t)] - \bar{Z}_{\ell,j,m}(t) \xrightarrow{M, K \rightarrow +\infty} 0.$$

Corollary 2 *The following holds in the large- (M, K) regime:*

$$\mathbb{E} \left[X_{j,m}^{(n)} \right] - \overline{X}_{j,m}^{(n)} \xrightarrow{M, K \rightarrow +\infty} 0, \quad (16)$$

$$\mathbb{E} \left[Z_{\ell,j,m}^{(n)} \right] - \overline{Z}_{\ell,j,m}^{(n)} \xrightarrow{M, K \rightarrow +\infty} 0 \quad (17)$$

where $\overline{X}_{j,m}^{(n)}$ and $\overline{Z}_{\ell,j,m}^{(n)}$ are the derivatives of $\overline{X}(t)$ and $\overline{Z}_{\ell,j,m}(t)$, respectively, at $t = 0$.

The deterministic quantities $\overline{X}_{j,m}^{(n)}$ and $\overline{Z}_{\ell,j,m}^{(n)}$ are functions of the p th derivatives $\delta_{\ell,k}^{(p)}$ and $\mathbf{T}_{\ell}^{(p)}$ of $\delta_{\ell,k}(t)$ and $\mathbf{T}_{\ell}(t)$ at $t = 0$, $p = 0, \dots, n$. Denote by $\overline{X}_{j,m}^{(0)} = \frac{1}{K} \text{tr}(\Phi_{j,j,m})$ and $\overline{Z}_{\ell,j,m} = \frac{1}{K} \text{tr}(\mathbf{R}_{\ell,j,m})$. We can compute the deterministic sequences $\overline{X}_{j,m}^{(n)}$ and $\overline{Z}_{\ell,j,m}^{(n)}$ as

$$X_{j,m}^{(n)} = - \sum_{k=1}^n \binom{n}{k} k \overline{X}_{j,m}^{(k-1)} \delta_{j,m}^{(n-k)} + \delta_{j,m}^{(n)}$$

$$\begin{aligned} \overline{Z}_{\ell,j,m}^{(n)} &= \frac{1}{K} \text{tr} \left(\mathbf{R}_{\ell,j,m} \mathbf{T}_{\ell}^{(n)} \right) - \sum_{k=0}^n k \binom{n}{k} \delta_{\ell,m}^{(n-k)} \overline{Z}_{\ell,j,m}^{(k-1)} \\ &+ \sum_{k=0}^n k \binom{n}{k} \delta_{\ell,m}^{(n-k)} \frac{1}{K} \text{tr} \left(\mathbf{R}_{\ell,j,m} \mathbf{T}_{\ell}^{(k-1)} \right) \end{aligned}$$

where $\delta_{\ell,k}^{(p)}$ and $\mathbf{T}_{\ell}^{(p)}$ can be computed using the iterative algorithm in [10]. Substituting the deterministic equivalent of Theorem 1 into (14) and (15), we get the following result.

Corollary 3 *Let $\overline{\mathbf{a}}_{j,m}$ and $\mathbf{B}_{\ell,j,m}$ be given by*

$$\begin{aligned} [\overline{\mathbf{a}}_{j,m}]_n &= \frac{(-1)^n}{n!} X_{j,m}^{(n)}, \\ [\mathbf{B}_{\ell,j,m}]_{n,p} &= \frac{(-1)^{n+p+1}}{(n+p+1)!} \overline{Z}_{\ell,j,m}^{(n+p+1)}. \end{aligned}$$

Then, $\mathbf{a}_{j,m}$ and $\mathbf{B}_{\ell,j,m}$ converge as

$$\max_{\ell,j,m} \left(\mathbb{E} \left[\|\overline{\mathbf{B}}_{\ell,j,m} - \mathbf{B}_{\ell,j,m}\| \right], \mathbb{E} \left[\mathbf{a}_{j,m} - \overline{\mathbf{a}}_{j,m} \right] \right) \xrightarrow{M, K \rightarrow +\infty} 0.$$

in the large- (M, K) regime and the SINRs converge as

$$\gamma_{j,m} - \overline{\gamma}_{j,m} \xrightarrow{M, K \rightarrow +\infty} 0 \quad (18)$$

$$\text{where } \overline{\gamma}_{j,m} = \frac{\mathbf{w}_j^H \overline{\mathbf{a}}_{j,m} \overline{\mathbf{a}}_{j,m}^H \mathbf{w}_j}{\sum_{\ell=1}^L \mathbf{w}_{\ell}^H \mathbf{B}_{\ell,j,m} \mathbf{w}_{\ell} - \mathbf{w}_j^H \overline{\mathbf{a}}_{j,m} \overline{\mathbf{a}}_{j,m}^H \mathbf{w}_j}.$$

5. SYSTEM PERFORMANCE OPTIMIZATION

In the previous section, we derived deterministic equivalents of the SINR at each UT in the multi-cell system as a function of the polynomial coefficients $\{w_{\ell,j}, \ell \in [1, L], j \in [0, J_{\ell} - 1]\}$ of the TPE precoding. These coefficients can be selected to maximize any system performance metric. Furthermore,

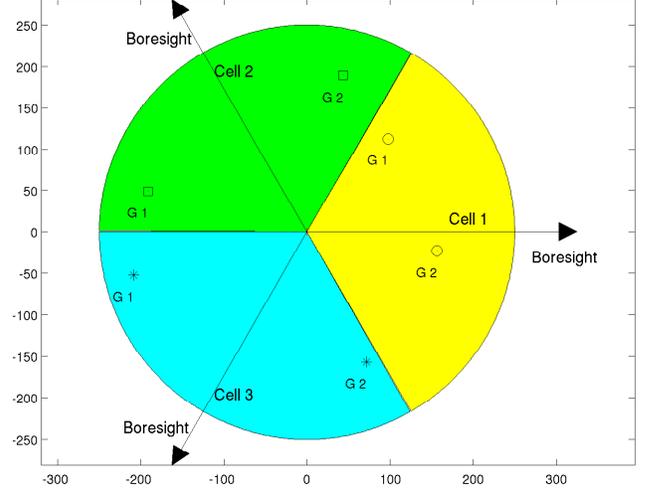


Fig. 1. Illustration of the three-sector site deployment with $L = 3$ cells considered in the simulations.

these coefficients need to be scaled to satisfy the transmit power constraints

$$\frac{1}{K} \text{tr} \left(\mathbf{G}_{\ell, \text{TPE}} \mathbf{G}_{\ell, \text{TPE}}^H \right) = \mathbf{w}_{\ell}^T \mathbf{C}_{\ell} \mathbf{w}_{\ell} = P_{\ell} \quad (19)$$

where \mathbf{C}_{ℓ} is a $J_{\ell} \times J_{\ell}$ matrix with elements given by

$$[\mathbf{C}_{\ell}]_{n,m} = \frac{1}{K} \text{tr} \left(\frac{\widehat{\mathbf{H}}_{\ell, \ell} \widehat{\mathbf{H}}_{\ell, \ell}^H}{K} \right)^{n+m+1}, \quad n, m \in [0, J_{\ell} - 1]. \quad (20)$$

We would like to pre-optimize the weights offline, thus the weights should not depend on the instantaneous value of the channel, but only its statistics. To this end, we substitute (19) by its asymptotic approximation

$$\mathbf{w}_{\ell}^T \overline{\mathbf{C}}_{\ell} \mathbf{w}_{\ell} = P_{\ell} \quad (21)$$

where $[\overline{\mathbf{C}}_{\ell}]_{n,m} = \frac{(-1)^{n+m+1}}{(n+m+1)!} \frac{1}{K} \text{tr} \left(\mathbf{T}_{\ell}^{(n+m+1)} \right)$.

In this paper, the performance metric is weighted max-min fairness. In other words, we maximize the minimal value of $\frac{\log_2(1+\gamma_{j,m})}{\nu_{j,m}}$ for some user-specific weights $\nu_{j,m}$. If we would like to mimic the performance of RZF, then $\nu_{j,m}$ should be the rate that user m in cell j achieves asymptotically with RZF precoding. Using deterministic equivalents, the corresponding optimization problem is

$$\begin{aligned} \max_{\mathbf{w}_1, \dots, \mathbf{w}_L} \min_{\substack{j \in [1, L] \\ m \in [1, K]}} & \frac{\log_2 \left(1 + \frac{\mathbf{w}_j^H \mathbf{a}_{j,m} \mathbf{a}_{j,m}^H \mathbf{w}_j}{\sum_{\ell=1}^L \mathbf{w}_{\ell}^H \mathbf{B}_{\ell,j,m} \mathbf{w}_{\ell} - \mathbf{w}_j^H \mathbf{a}_{j,m} \mathbf{a}_{j,m}^H \mathbf{w}_j} \right)}{\nu_{j,m}} \\ \text{subject to} & \quad \mathbf{w}_{\ell}^T \overline{\mathbf{C}}_{\ell} \mathbf{w}_{\ell} = P_{\ell}, \quad \ell \in [1, L]. \end{aligned} \quad (22)$$

This problem is non-convex, but very similar to the multi-cast beamforming problems analyzed in [12]. In particular, we can instead solve the following tractable relaxed convex problem:

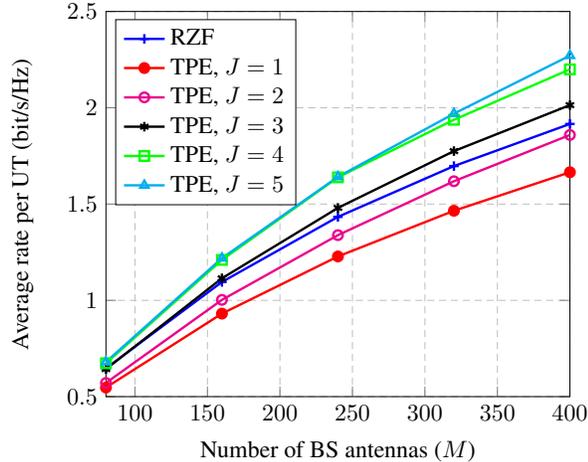


Fig. 2. Comparison between conventional RZF precoding and the proposed TPE precoding with different orders $J = J_j \forall j$.

$$\begin{aligned} & \max_{\mathbf{w}_1, \dots, \mathbf{w}_L, \xi} \xi & (23) \\ \text{subject to } & \mathbf{W}_\ell \succeq \mathbf{0}, \text{tr}(\bar{\mathbf{C}}_\ell \mathbf{W}_\ell) = P_\ell, \ell \in [1, L] \\ & \frac{\bar{\mathbf{a}}_{j,m}^H \mathbf{W}_j \bar{\mathbf{a}}_{j,m}}{\sum_{\ell=1}^L \text{tr} \bar{\mathbf{B}}_{\ell,j,m} \mathbf{W}_\ell - \bar{\mathbf{a}}_{j,m}^H \mathbf{W}_j \bar{\mathbf{a}}_{j,m}} \geq 2^{\nu_{j,m} \xi} - 1, \forall j, m. \end{aligned}$$

6. SIMULATION RESULTS

In this section, we consider the three-site sector in Fig. 1. Each of the three BSs is equipped with an horizontal linear array of M antennas. The radiation pattern of each antenna is

$$A(\theta) = -\min\left(12\left(\frac{\theta}{\theta_{3\text{dB}}}\right)^2, 30\right) \text{ dB}, \theta_{3\text{dB}} = 70^\circ.$$

We assume that the UTs are divided into $G = 2$ groups as described in [11]. The pathloss between UT m in the group g and cell ℓ is selected as in [11]: $\text{PL}(d_{j,m}) = 1/(1 + (\frac{d_{j,m}}{d_0})^\delta)$, where $\delta = 3.7$ is the pathloss exponent and $d_0 = 30$ m is the reference distance. We use the channel covariance model in (3), but scale it by the pathloss and antenna radiation pattern.

For simplicity, we let the effective pilot SNR, ρ_{tr} , and all the downlink SNRs, $\rho_{\text{dl}} = \frac{P_\ell}{\sigma^2}$, be 15 dB. We compare the average user throughput, $\frac{1}{KL} \sum_{j=1}^L \sum_{m=1}^K \mathbb{E}[\log_2(1 + \gamma_{j,m})]$, of the proposed TPE precoding with that of conventional RZF precoding. The coefficients of TPE precoding are achieved by solving the relaxed problem in (23) offline. Using Monte-Carlo simulations, Fig. 2 shows the performance for $K = 40$ users and different number of antennas at each BS: $M \in \{80, 160, 240, 320, 400\}$. Unlike the single-cell case analyzed in [1], where TPE precoding is merely an approximation of RZF precoding, we see that TPE precoding achieves higher user rates for all $J_j \geq 3$ in the multi-cell case. This can be attributed to the tractable offline optimization of the polynomial coefficients which allows for a better coordination of intra-cell and inter-cell interference, a feature which could not be implemented for the RZF precoding.

7. CONCLUSION

In this paper, we generalize the low-complexity TPE precoding family from [1] to multi-cell scenarios. In particular, we derive deterministic equivalents for the asymptotic SINRs in massive MIMO systems where the number of antennas and users grow large with a fixed ratio. The most interesting feature of these expressions is that they only depend on the channel statistics and not on the instantaneous channel realizations. This enables us to optimize TPE precoding in an offline manner. The performance of the proposed precoding method is illustrated using simulations. We note that contrary to the single-cell case, TPE precoding outperforms the RZF precoding in terms of both complexity and throughput.

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